SCS139 HW2

Saturday, February 02, 2013 10:20 PM

Q1 $\underset{\substack{\uparrow \\ 3}}{i_2 = \alpha} \underset{\substack{\uparrow \\ 3.61\times10^{-3}\,A}}{i_1}$, $d = 16\text{ cm} = 0.16\text{ m}$

Recall that a magnetic field $\vec{B}$ due to a current i in a long straight wire is given by

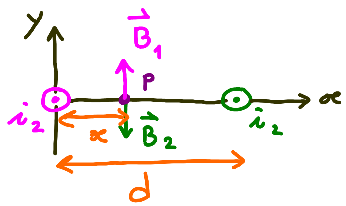
$$B = \frac{\mu_0 i}{2\pi R}.$$

R ← perpendicular distance from the wire

Its direction can be determined by the curled-straight right-hand rule.

i, $\vec{B}$ line

Consider a point P on the x axis.



Applying the rule, we see that the two parallel currents create "fighting" magnetic fields at point P.

The net magnetic field is required to be 0 at P.

Therefore, we need $B_1 = B_2$

$$\Downarrow$$

$$\frac{\cancel{\mu_0} i_1}{2\cancel{\pi} x} = \frac{\cancel{\mu_0} i_2}{2\cancel{\pi}(d-x)}$$

$$\Downarrow$$

$i_1 d - i_1 x = i_2 x$

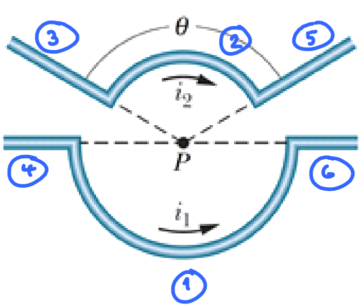
$$x = \frac{i_1}{i_1+i_2} d = \frac{i_1}{i_1+\alpha i_1} d = \frac{1}{1+\alpha} d$$

(a) $\alpha = 3 \Rightarrow x = \frac{1}{1+3} d = \frac{1}{4} 0.16 = 0.04\text{ m} = 4\text{ cm}$

(b) When i_1 and i_2 are both doubled, the ratio $\frac{i_2}{i_1}$ is still $\alpha = 3$.

Therefore, the zero-field point remains unchanged.

Q2 Our strategy is to break the given shape into segments that we have formulas for finding the magnetic fields.



In this problem, we break the given shape into six segments as shown in the figure.

There are two circular arcs (1 and 2) and four radial segments (3, 4, 5, and 6)

The net magnetic field $\vec{B}_{net}$ at P is simply the (vector) sum of all individual contributions from the segments.

Recall that radial segments (3, 4, 5, and 6) do not contribute any magnetic field. Therefore, we only need to focus on the circular arcs.

① and ② are circular arcs whose magnetic fields at the center can be found by

$$B = \frac{\mu_0 i}{2R}\underbrace{\left(\frac{\theta}{2\pi}\right)}_{} = \frac{\mu_0 i}{2R}\underbrace{\left(\frac{\theta}{360^\circ}\right)}_{}$$

in radians — in degree

fraction of a complete circle.

For ②, fraction $= \frac{120^\circ}{360^\circ} = \frac{1}{3}$

For ①, fraction $= \frac{180^\circ}{360^\circ} = \frac{1}{2}$

(a)

The directions of the magnetic fields at center of circular arcs are determined by the curled-straight right-hand rule. Applying the rule, we found that, at P

arc ② gives $\vec{B}_2$ which points into the page and

arc ① gives $\vec{B}_1$ which points out of the page.

Also, $B_1 = \frac{\mu_0 i_1}{2 r_1}\left(\frac{1}{2}\right) = 2\mu_0$ (i_1 = 0.4 A, r_1 = 0.05 m) $\quad B_2 = \frac{\mu_0 i_2}{2 r_2}\left(\frac{1}{3}\right) = \frac{10}{3}\mu_0 \approx 3.3\mu_0$ ($i_2 = 2 \times i_1 = 0.8$ A, r_2 = 0.04 m)

Therefore, $B_{net} = B_2 - B_1 = \frac{4}{3}\mu_0 \approx 1.68\ \mu T$.

(b) Because $B_2 > B_1$, $\vec{B}_{net}$ is into the page.

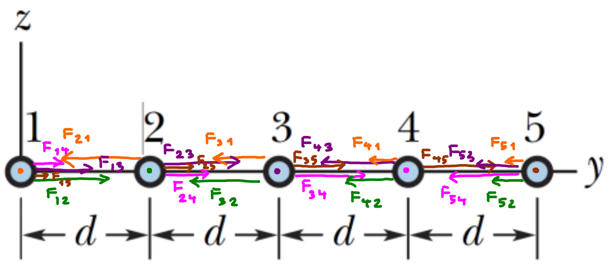
For parts (c) and (d), the current i_1 is reversed. Therefore $\vec{B}_1$ now points into the page.

(c) $\vec{B}_1$ and $\vec{B}_2$ are now in the same direction. Therefore,

$$B_{net} = B_1 + B_2 = \frac{16}{3}\mu_0 \approx 6.7\ \mu T.$$

(d) The direction of $\vec{B}_1$ and $\vec{B}_2$ are both into the page. Therefore, the direction of $\vec{B}_{net} = \vec{B}_1 + \vec{B}_2$ is also into the page.

Q3



Recall that

1) $F_{ba} = \frac{\mu_0 L i_a i_b}{2\pi D}$

2) Two wires with parallel current attract each other.

Here, $i_a = i_b = i = 3A$.

The distance between wire "a" and wire "b" is given by $D = d|a-b|$

(a) $\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \vec{F}_{15} = \frac{\mu_0 L i^2}{2\pi}\left(\frac{1}{d}\hat{j} + \frac{1}{2d}\hat{j} + \frac{1}{3d}\hat{j} + \frac{1}{4d}\hat{j}\right) = \frac{\mu_0 L i^2}{2\pi d} \times \frac{25}{12}\hat{j} \approx 4.7 \times 10^{-4}\,\hat{j}\ N$

(b) $\vec{F}_2 = \vec{F}_{21} + \vec{F}_{23} + \vec{F}_{24} + \vec{F}_{25} = \frac{\mu_0 L}{2\pi} i^2\left(\frac{1}{d}(-\hat{j}) + \frac{1}{d}\hat{j} + \frac{1}{2d}\hat{j} + \frac{1}{3d}\hat{j}\right) = \frac{\mu_0 L i^2}{2\pi d} \times \frac{5}{6}\hat{j} \approx 1.9 \times 10^{-4}\,\hat{j}\ N$

(a) $F_1 = F_{12} + F_{13} + F_{14} + F_{15} = \frac{[illegible]}{2\pi}\left(\frac{1}{d}\hat{j} + \frac{1}{2d}\hat{j} + \frac{1}{3d}\hat{j} + \frac{1}{4d}\hat{j}\right) = \frac{[illegible]}{2\pi d} \times \frac{[illegible]}{12}\hat{j} \approx$ [illegible] $\hat{j}$ N

(b) $\vec{F}_2 = \vec{F}_{21} + \vec{F}_{23} + \vec{F}_{24} + \vec{F}_{25} = \frac{\mu_0 L}{2\pi} i^2 \left(\underbrace{\frac{1}{d}(-\hat{j}) + \frac{1}{d}\hat{j}}_{\text{cancel}} + \frac{1}{2d}\hat{j} + \frac{1}{3d}\hat{j} \right) = \frac{\mu_0 L}{2\pi d} i^2 \times \frac{5}{6}\hat{j} \approx 1.9 \times 10^{-4}\,\hat{j}$ N

(c) $\vec{F}_3 = \vec{F}_{31} + \underbrace{\vec{F}_{32} + \vec{F}_{34}}_{\text{cancels}} + \vec{F}_{35} = 0$ N

cancels

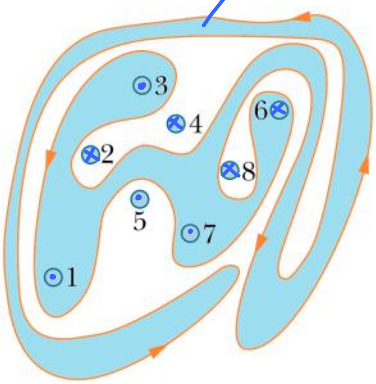
(Observe that $\vec{F}_{31} = -\vec{F}_{35}$ and $\vec{F}_{32} = -\vec{F}_{34}$

(d) $\vec{F}_4 = -\vec{F}_2 \approx -1.9 \times 10^{-4}\,\hat{j}$ N

(e) $\vec{F}_5 = -\vec{F}_1 \approx -4.7 \times 10^{-4}\,\hat{j}$ N

Q4

The shaded area shows the enclosed area by the loop.



Observe that only i_1, i_3, i_6, i_7 are enclosed.

note the different sign because i_6 has different direction than other currents

So, $i_{enc} = i_1 + i_3 - i_6 + i_7 = 1i + 3i - 6i + 7i = 5i$ $\;\; i = 4.5$ mA

$= 22.5$ mA

Applying Ampere's law, we get

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc} = 90\pi \times 10^{-10} \approx 2.83 \times 10^{-8}\ \text{T}\cdot\text{m}$$

$4\pi \times 10^{-7}\ \text{T}\cdot\frac{\text{m}}{\text{A}}$